METAL STRING

PHYSICAL MODELING OF BOWED STRINGS – A NEW MODEL AND ALGORITHM

PREPRINT

Marco Palumbi, Lorenzo Seno
marco.palumbi@bigfoot.com, lorenzo.seno@bigfoot.com

Centro Ricerche Musicali
Via Lamarmora, 18 - 00185 Roma - Italia
crm.it@usa.net
http://www.axnet.it/crm - http://www.geocities.com/Vienna/4934

ABSTRACT

The simulating model and algorithm here discussed are based upon a method quite different from the well know wave-guide approach – the currently used model in musical research.
Our approach is, at present, more computationally complex, but the control of the various physical parameters and of their meaning is very straightforward. Furthermore the algorithm makes no assumption on the time invariance of the system, so the variation of any parameter does not introduce artifacts.
Our model implements both the viscous friction of the string with the air, and the internal friction – i.e. the energy dissipation due to internal viscous-like behavior of the string.
The friction of the bow is represented by a discontinuous function, which simulates the thermal behavior of the rosin by means of an hysteresis mechanism, and models also the roughness of the bow by means of white noise.
The time-varying controllable parameters are the tension/density ratio of the string, the two friction coefficients, the speed and the pressure of the bow, and the (since now discrete) point of bowing on the string *(b)*
The computation algorithm here discussed is an intermix of a method similar to the finite element one for what concerns the integration over the space, and to the finite difference method for what concerns the integration of the evolution of the system.
Our model can be used to play in extreme parametric conditions, beyond the bowed strings performance tradition.
Although our model fully neglects twisting and longitudinal motions, as well as the bridge admittance, it produces very likely sounds. One can maybe infer that these characteristics are less important than one may suspect.
Using this model, the Italian composer Michelangelo Lupone wrote the tape part of the string quartet "Corda di metallo" ("Metal string"), whose first world performance was held in Rome in 1997 by the Kronos Quartett.

FOREWORD

We started in the autumn of 1996 our studies and researches about physical modeling. The results of our readings of the related literature was that the prevailing approach was the wave-guide. On the other hand, in the literature on the physics of the violin (starting from Raman, Cremer, et al.) was not clearly stated the importance of the various factors in the final behavior of the bowed string instruments. At that time was also available to us a powerful computing system, based on a parallel architecture of DSP Texas C40.
Under these circumstances we believed that an approach not so sensitive to the computational cost was suitable, favoring instead a direct relationship between the parameters of the model and the physics of the phenomena. We hoped that this approach would make easier the learning of the importance of the various

physical factors in the sound of the bowed strings. For us, in the evaluations of the sounds, the musical point of view was the most important one.
Because of our personal tendency, and because of our bonds to contemporary composers, we were more interested in the search of new interesting sounds – hardly obtainable within the bounds of the actual physical world - than in the imitation of true, "correctly" played, instruments.
Our model (in the form of a computer software) was actually used by the composer Michelangelo Lupone to get suggestions about new performing techniques and to compose the magnetic tape of the "Metal string" quartet (from whom the title of this paper) for strings, tape and spatializer (Kronos Quartett, Rome, 1997).

THE MODEL

The string

Our PDE of the free string is:

$$\frac{d^2}{dt^2} y(x,t) = \frac{T}{\mu} \cdot \frac{d^2}{dx^2} y(x,t) - S \cdot \frac{d}{dt} y(x,t) - S_i \cdot \frac{d^2}{dx^2} \frac{d}{dt} y(x,t)$$

with boundary conditions:

$$y(0,t) = y(L,t) = 0$$

Where:

T	(Newton)	tension of the string.
μ	(Kg/m)	linear density of the string.
S	(sec^{-1})	coefficient due to the viscous friction with air.
S_i	(m^2/sec)	coefficient due to the viscous internal friction.

A few words on the presence (and the absence) of some terms. The classical dispersive term is absent:

$$\frac{d^4}{dx^4} y(x,t)$$

This term is due to the stiffness of the string. It is responsible of the dependence of the propagation speed on the frequency. Because of these different speeds, partials are in super-harmonic frequency relationship.
This has important effects on the timbre of the sounds, especially with stiff strings –e.g. like in the low section of the piano.
Strings used in bowed instruments (as in the violin family) have low stiffness, but many researchers believe it is the reason of the so-called rounding effect which can be easily observed by means of experiments
Other researchers (Woodhouse, 1992) suggested that this effect is due, for the most part, to the action of the bow, particularly to the hysteresis in the friction behavior of the rosin.
We skipped that term mainly because our integration method introduces at present some computational error whose effect is to move the partial frequencies away from each other – the inclusion of this term in the computation being a straightforward task.
A few words now about the last, right-hand, mixed term, generally neglected in the literature. It represent the effect of the energy losses due to internal, viscous, friction which offer resistance to changes in the curvature.
This phenomenon is responsible of a main behavior of actual, free running strings: the higher the frequency partial, the faster its dumping. For instance, if you pluck a string, during the transient you can hear many partial – a rich sound. Instead in the release part of the sound you can hear just the fundamental.

The pitch of the sound, in our model, is due to the parameter T/μ and/or to the boundary conditions (specifically, to the parameter L). You can thus obtain a variation of the pitch either varying T/μ or varying the length L of the string – two independent ways to obtain the same effect.

Varying the tension corresponds to physically twisting the tuning peg. Length variation is quite similar to the action of the left-hand of the performer.
In both circumstances, the system (and the corresponding PDE) is time-variant. To avoid artifacts, the integration method must not make any implicit assumption on time-invariance. The way we integrate the evolution of the system – a finite difference method - does not make actually any assumption about the time-invariance of T/µ.. Thus, you can modify in any desired way this parameter. Varying the length (a spatial parameter) sets some subtle question: you must correctly transfer the final to the initial condition (the shape of the string) from one length to the next one. We have not yet implemented this feature.

The stimulus

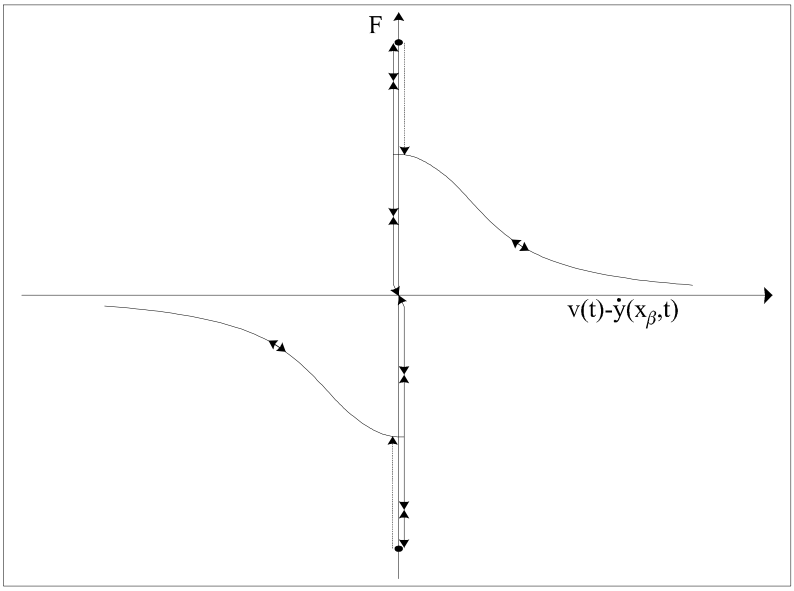
The excitation of a string motion is a quite different problem, depending on the way you want to excite it: plucking, hitting or bowing.
At a first glance you can consider the excitation as a time-varying function applied to a specific point x_β of the string in the wise of a force, or a boundary condition for the speed or the displacement y.
A more realistic approach requires the consideration of the interactions between the excitation function and the string – which requires the introduction of a term which is function of both the excitation and some state variable of the string.
This is the case of the exciting behavior of the bow, which deeply interacts with the string, and introduces into the motion equation a non-linear term, which depends on the true, external, exciting functions (i.e. pressure and speed of bowing) and state variables of the string. In this way, we get a system that is both non-linear and time-variant.
In our model we consider as excitation functions the pressure and the speed of the bow, and as interacting state variable the location of bowing point x_β, the speed and the acceleration of the bowing point.
The term so introduced has the form:

$$F\left(v(t), p(t), x_\beta(t), \frac{d}{dt} y\left(x_\beta(t), t\right), x, \sigma(t)\right)$$

where:

v(t)	(m/sec)	Speed of the bow
p(t)	(Newton)	Pressure of the bow
$x_\beta(t)$	(m)	Bowing point location.
$\sigma(t)$		Boolean state variable depending on the “stick” aut “slip” condition of the bowing point.

Our bow is in some way quite different from that usually implemented by means of continuous speed/force functions.
During the stick state, the bowing point is glued to the bow, and we force the speed of that point to be equal to the bow speed (i.e. the relative speed is zero), by applying a suitable acceleration to the point x_β.
During the slip stage, F is taken as a component of the acceleration applied to the point (the acceleration is the force / linear density), due to a (non linear) function of the mutual speed of the point and the bow.
In both cases, the exciting term is applied to a single point, and at present this point must be one of the N regularly spaced points of the spatial sampling of the string shape (see below). So we have until now a discrete bowing point position.

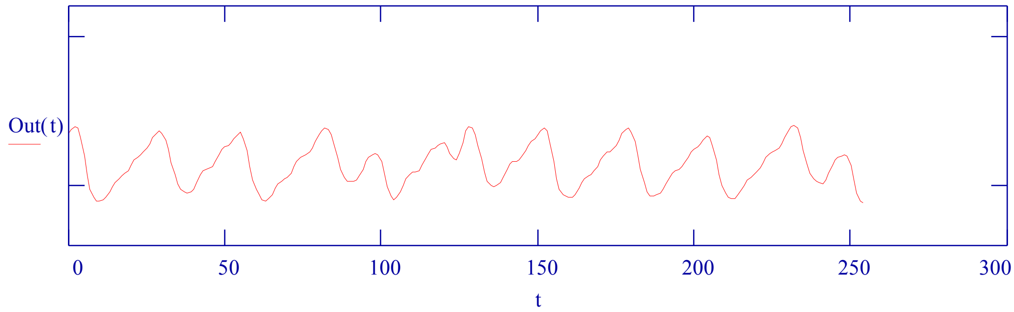


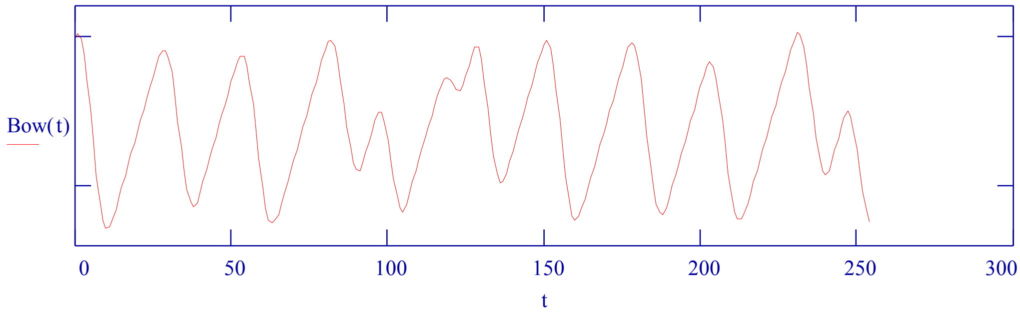
State diagram of the bowing point. Acceleration (force / linear density) versus point-bow relative speed.

As you can see, the "stick"→"slip" transition has a threshold that is twice the "slip"→"stick" transition (which happens for zero relative speed).
The effect of the bowing pressure is to linearly scale the acceleration (both threshold and continuous curve). One may explore other kinds of dependency.
We doesn't take explicitly into consideration the temperature of the point - a variable affecting the fluidity and thus the friction behavior of the rosin (a natural polymer with a complex behavior). But the threshold emulates the cooling effect of the rosin during the stick time, during which the dissipation is zero.
The noise due to the rubbing of the bow over the string is modeled by means of white noise added to the F curve, in such a way to preserve the curve as a "mean value" of the random value. In this way we non only "add" noise to the sound (proportional to the pressure), but we impart a "chaotic" behavior due to the random interaction between bow and string.

The sound

To produce the sound, we take the displacement of the spatial sampling point nearer to the bridge (the point number 1, 0 being the origin, i.e. the bridge itself). One can think of this point as a "secant" approximation of the tangent to the string in the bridge position (i.e. the first spatial derivative in the origin), being the tangent the expression of the strain against the bridge.
It is not hard to implement more exact behaviors, as f.i. the admittance of the bridge.
The waveforms show the typical sawtooth shapes. Here you can see a sample of the displacement in the bowing point (Bow), and in the output point (Out). Abscissa shows the time in samples @ 44.1 KHz.





THE METHOD OF CALCULUS

The method of calculus, for the integration of the spatial variable, is similar to the finite elements one (in the sense that it is based on the decomposition of the unknown solution in a series of orthonormal functions); while the method used for the integration of the system temporal evolution is a finite differences one.

The instantaneous shape of the string in the space is decomposed in a set of harmonic functions up to the N'th order that is the highest allowable spatial frequency for the string. This maximum spatial frequency is obviously related to the number of achievable partials, and indeed they'll be equal in the case of free evolution. Therefore N will be chosen as a function of the desired passband, remembering that the complexity of the algorithm increases with N.

Once the number of harmonics in the series has been fixed, one can represent, at a given instant, the shape of the string in the continuum by means of a finite number (equal to N) of samples of its position.

Let them be

$$\left(x_1, y_1\right), \left(x_2, y_2\right) .. \left(x_N, y_N\right)$$

It is possible to obtain, from these, the coefficients of the harmonic decomposition – e.g. extracting their DFT. Once the coefficients are gotten, the position of the string in the space is known analytically (in the whole interval 0-L, and not only in correspondence of the sampling positions). Thanks to the knowledge of an analytic form of the solution, we can easily derive its differential properties in the interval 0-L; in particular we can formally get the curvatures (i.e. second spatial derivatives) in each of the sampling points:

$$\frac{d^2}{dx^2} y\left(x_1\right), \frac{d^2}{dx^2} y\left(x_2\right) .. \frac{d^2}{dx^2} y\left(x_N\right)$$

Leaving out the boring algebra, the calculation of the curvatures can be performed through the product of a constant matrix with the vector of positions *y*.
At this point the equation of the string can be broken in N second order motion equations of the sampled points, in which the left term (acceleration) results from the curvature, the velocities (which appear in the term of viscous friction with air) and from the time variation of the curvature itself (term of internal viscous friction).
The velocity can be obtained by integration of acceleration, while for the calculation of time variation of the second derivatives, we store their previous values and substitute the derivative with the finite difference.
At this point the knowledge of acceleration allows the calculation, by means of integration, of the further position and velocity, and the calculation loop starts again.
This kind of integration in the time domain introduces an error that brings to a spread of the spectral lines; the partials now lie in superharmonic ratios: an effect that behaves like the stiffness of the string. To reduce this side effect we introduced a suitable oversampling.

RESULTS

The model has been widely tested and used with N=16 and a four times oversampling factor.
As to the excitations, we implemented simple models of pinch and percussion that brought very satisfactory results from the standpoint of imitation.
We made simulations with the specific goal to imitate sound emissions of actual bowed instruments and they gave encouraging results in the case of little variations of the pitch, particularly as to the bow attack transients and the evolution of long-lasting sounds. Furthermore, the sounds generated with our model approximate very well, from an acoustic standpoint, the actual ones, especially in the variation of pressure, velocity and position also in non-standard conditions of execution.
The execution of musical phrases needs, for any kind of articulation, the correct modeling of the variation of the string length L, which represents a boundary condition.
Pitch variations obtained through the variation of the string tension are already feasible, but, in this case, the transients sound quite different from those obtained varying the string length.

GUIDELINES FOR FURTHER RESEARCH

The work we have done until now, is nothing but a starting point. There are many directions for further investigations both in the field of model improvement – e.g. to include other musically relevant issues that have not been implemented yet, and to increase the efficiency and the accuracy of the algorithm.

The model

The first feature that must be implemented is the possibility to dynamically vary the string length instead of tension, to perform pitch variations.
Many hints suggest a greater, or maybe better simulated, rounding effect. The ways of further investigation are fundamentally two: the introduction of the term which models the stiffness of the string, and the simulation of the thermic hysteresis of the rosin. It is possible – and in the present model quite easy to implement – to introduce the temperature of the bowing point as a state variable and the dependence on it of the friction coefficient of the rosin.
Other investigations can be made about the points of constraint: on one side the fingers of the left hand with their damping features; on the other side, the bridge with its complex admittance whose role in the definition of the timbre and in the dynamic of the string is not so clear yet.
Furthermore we could improve our algorithm allowing the continuous variation of the bowing point which is now constrained to be one of the spatial sampling points; this would imply a much greater expense of computation that could probably be compensated by more efficient computation techniques (see. beyond).
The finite dimension of the bow could be modeled considering that in the “stick” phase it produces a shortening of the string or applying it to a couple of adjacent points.
Further improvements can be introduced in the spectral modeling of the bow-rubbing noise. Since now we have considered it as a white noise, but in the reality it depends on the velocity of the bow; thus a non-

white spatial noise, centered around a frequency and brought back in the time domain through the velocity, might be a more suitable model.

The algorithm

At present the computational cost is $O(N^2*S)$, where N is the number of spatial sampling points and S is the temporal oversampling factor. This law assumes the time invariability of the spatial sampling points.
The algorithm is performed in about twice of the real time on a Pentium® 133, with N=16 and S=4.
Different approaches, based on fast FFT-like algorithms, should lead to a computational cost of the kind O(N*ld(N)*S) or even better. We are also studying the use of Winograd algorithms; they should imply N to be a prime number, but this wouldn't be a limitation in our context.
The individuation of a fast DFT algorithm (in the wise of FFT) performing non-uniform and real-time sampling for the decomposition in spatial harmonics – and the consequent inversion of the spectrum multiplied by ω^2 to obtain the local curvatures - would allow continuous values both for bowing and damping . This will allow the modeling of both the bow position (β) and width imposing the excitation on the bow boundaries and of light fingerings to obtain harmonics, or of techniques of length variation.
As to time integration we are studying fast convergence methods, to reduce the oversampling factor and the approximation error. Likely directions are Runge-Kutta method or fourth order prediction-correction (Hamming) method, that may lead to an error proportional to $(\Delta t)^5$, where Δt is the sampling period. Maybe these methods will allow S=2 or even S=1, with a reduction of two or four times of the calculation delay, making real-time feasible by now.
Furthermore, the algorithm is well suitable for parallelisation, once the most convenient topology has been chosen. Thus a multiprocessor system may allow, even in short times, a realistic polyphony (two-four voices).

CONCLUSIONS

In this, like in other fields, the evaluation of the various direction of research strongly depends on one's goals.
If we assume the imitation of actual instruments as a primary goal of the physical model simulation, we probably know how to interpret the results we obtained. But if our goal is to provide a new synthesis instrument which is both better controllable and more stimulating when compared to other methods, then the evaluation criteria become more complex and, in our opinion, depend strongly on musical considerations.
In this latter case the imitation of actual instruments is relevant but only in an indirect way. It is a guideline to verify the correctness of the hypothesis and the methods adopted, but is no longer, by its own, an interesting goal.
From the point of view of contemporary music, the modeling and implementation of features and behaving that are physically impractical may result more important and interesting even if unverifiable.
If we assume this point of view, the key of the evolution of our research lies in the hands of musicians and composers rather than in those of the researchers. Thus it's clear that such a research can be led but through a strong interaction between researchers and musicians.

REFERENCES

Woodhouse, J. 1992. "Physical Modeling of Bowed Strings." *CMJ*, 16,4, pp. 43-56

Smith, J.O. III 1996. "Discrete Time Modeling of Acoustic Systems with Applications to Sound Synthesis of Musical Instruments." *Proceedings of the Nordic Acoustical Meeting,* Helsinki, June 12-14 (http://www.hut.fi/TKK/Akustiikka/aku/man96/)

Smith, J.O. III 1996. "Physical Modeling Synthesis Update." *CMJ,* 20,2, Summer 1996 pp.44-56

Borin, G., De Poli, G, Sarti, A. 1992. "Algorithms and Structures for Synthesis Using Physical Models." *CMJ,* 16,4 Winter 1992

Karplus, K., Strong, A. 1983. "Digital Synthesis of Plucked String and Drum Timbres." *CMJ* 7,2 pp.43-55

Jaffe, D.A., Smith, J.O. III 1983. "Extension of the Karplus-Strong Plucked-String Algorithm." *CMJ* 7,2 Summer 1983

Chafe, C. 1989 "Simulating Performance on a Bowed Instrument." *Current Directions in Computer Music Research* - Edited by Matew, M. and Pierce, J. MIT Press 1989

Fletcher, N., Rossing, T.D., 1988. "The Physics of Musical Instruments." Springer, N.Y. ISBN 0-387-94151-7 ISBN 3-540-94151-7

Hutton, D.V., 1981. "Applied Mechanical Vibrations." McGraw-Hill 1981. ISBN 0-07-031549-3

Olson, H.F., 1952. "Music, Physics and Engineering." Dover 1967

Johnson, C., 1990. "Numerical solution of partial differential equations by finite element method." Cambridge University Press ISBN 0 521 345 146 ISBN 0 521 347 580.

Boyden, D.D., Monosoff, S., Schwarz, B., Marx, K., Slatford, R., Hutchins, C.M., et al. 1980. "Gli archi" Ricordi 1995 ISBN 88-7592-417-1 (Titolo originale: "The Violin family" Macmillan Press).

Chassery, J.M. 1991. "Transformation de Fourier discrète. Definition et aspects algorithmiques" in Baranger, J. "Analyse Numerique" Hermann. ISBN 2 7056 6093 2. Pp. 459-514

Florensm J-L., Cadoz, C. 1991. "The Physical Model: Modeling and Simulating the Instrumental Universe" *Representation of Musical Signals* edited by De Poli, G., Piccialli, A., Roads, G. MIT Press ISBN 0-262-04113-8 (hc) pp.227-268.

Adrien, J-M., 1991. "The Missing Link: Modal Synthesis" *Representation of Musical Signals* edited by De Poli, G., Piccialli, A., Roads, G. MIT Press ISBN 0-262-04113-8 (hc) pp.269-297

Selesnick, I.W., Sidney Burrus C., 1994. "Extending Winograd's Small Convolution Algorithm to Longer Lengths" *ISCAS Proceedings 1994*

Selesnick, I.W., Sidney Burrus C., 1995. "Automatic Generation of Prime Length FFT Programs" *IEEE Transactions on Signal Processing.*