METAL STRING

PHYSICAL MODELLING OF BOWED STRINGS – A NEW MODEL AND ALGORITHM

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Abstract

The paper introduces a new model of a bowed string, whose simulating algorithm is based upon a method quite different from the well know wave-guide approach – the currently used one in today musical research.
Our model implements both the viscous friction of the string with the air, and the internal friction, and a discontinuous bow, which includes also a parametric noise model (i.e., the noise is not simply added to the sound). The approach brings to an inherently time-variant system, so player can freely change any parameter without artifacts. The continuously controllable parameters are the tension/density ratio of the string (i.e- the pitch of the note), the two friction coefficients, the speed and the pressure of the bow. In today implementation, the bowing point (b) is variable but discrete.
Using a software based on our model, the Italian composer Michelangelo Lupone wrote the tape part of the string quartet "Corda di metallo" ("Metal string") (Kronos Quartett - Rome 1997).

1 INTRODUCTION

Musical Research in the field of musical instruments synthesis by Physical modeling is dominated by the tendency to use delay lines (or waveguides) as a solving algorithm [4][5][6][10]. This is mainly because of the low computational cost of delay lines, and may be because it is a form of historic tribute to the first method invented -- the Karplus-Strong algorithm [7][8]. Our approach is instead an evolution of the finite difference method, as an attempt to bypass its limitations. Finite difference method is equivalent to a physical model of a discrete mass-spring system, which inherently brings to partials with sub-harmonic ratios. Our string model is continuous, thus not suffering this inconvenient.
Non only the methods, by also the goals of our work are quite different from the prevailing one in this research field. Because of our personal tendency, and because of our bonds to contemporary composers, we were more interested in the search of new interesting sounds rather than in the imitation of true, traditionally played, instruments.
Our model (in the form of a computer software) was in effect used by the composer Michelangelo Lupone to get suggestions about new performing techniques and to compose the magnetic tape part of the "Metal string" quartet (from whom the title of this paper) for strings, tape and spatialiser (Kronos Quartett, Rome, 1997).

2 THE MODEL

2.1 The string

Without loss of generality, consider a string of unit length, whose free PDE is:

$$\frac{d^2}{dt^2} y(x,t) = \left(\frac{T}{\mu}\cdot\frac{d^2}{dx^2} y(x,t) - S\cdot\frac{d}{dt} y(x,t)\right) \ldots$$
$$+ -S_i\cdot\left(\frac{d^2}{dx^2}\frac{d}{dt} y(x,t)\right)$$

with boundary conditions:

$$y(0,t) = y(1,t) = 0$$

Where:

T	(Newton)	tension of the string.
μ	(Kg/m)	linear density of the string.
S	(sec^{-1})	coefficient due to the viscous friction with air.
S_i	(m^2/sec)	coefficient due to the viscous internal friction.

A few words about the presence (and the absence) of some terms. The classical dispersive term is absent:

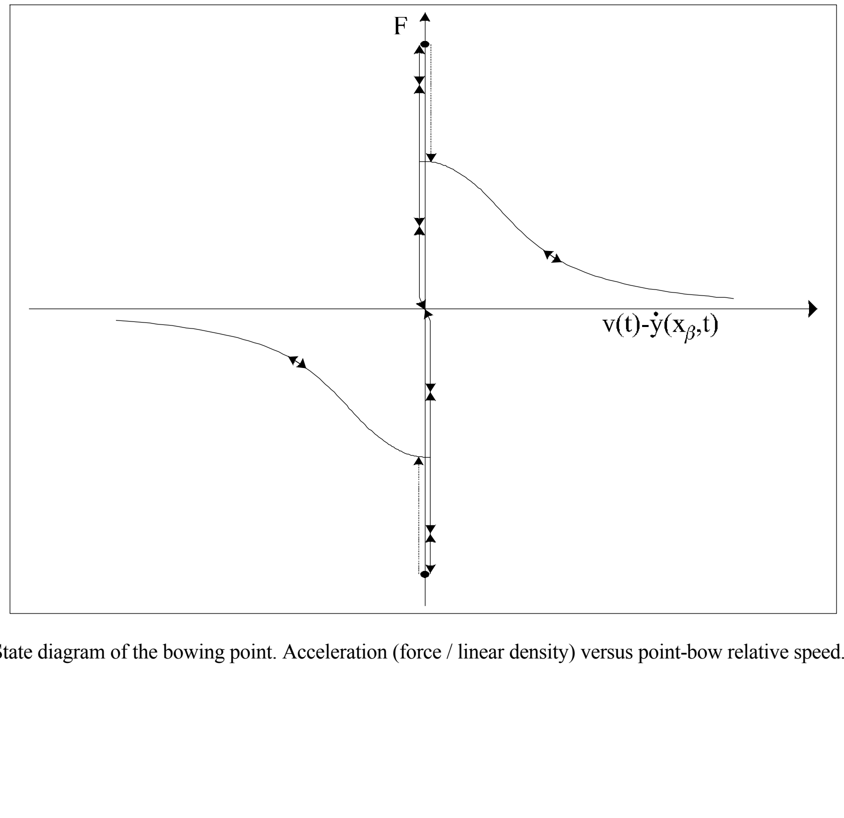
$$\frac{d^4}{dx^4} y(x,t)$$

This term is due to the stiffness of the string. It is responsible of the dependence of the propagation speed

on the frequency. Because of these different speeds, partials are in super-harmonic frequency relationship.
This has important effects on the timbre of the sounds, especially with stiff strings –e.g. like in the low section of the piano.
Strings used in bowed instruments (as in the violin family) have low stiffness, but some researchers believe it is the reason of the so-called "rounding effect" [1][2]. Other researchers [3] suggested that this effect is due, for the most part, to the action of the bow, particularly to the hysteresis in the friction behavior of the rosin.
We skipped that term mainly because our integration method introduces at present some computational error whose effect is to slightly move the partial frequencies away from each other – the inclusion of this term in the computation being a straightforward task. On the other hand, strings used in the violin family have normally very low stiffness.
A few words now about the last, right-hand, mixed term generally neglected in the literature. It represents the effect of the energy losses due to internal, viscous, friction, which offer resistance to changes in the curvature. This phenomenon is responsible of a main behavior of actual, free running strings: the higher the frequency partial, the faster the dumping. For instance, if you pluck a string, during the transient you can hear many partials – a rich sound. Instead in the release part of the sound you can hear just the fundamental.
The pitch of the sound is due to the parameter T/μ. The way we integrate the evolution of the system – a finite difference method - does not make actually any assumption about the time-invariance of T/μ nor of any other parameter like internal and external dumping. You can thus modify in any desired way these parameters without any artefact.

2.2 *The stimulus*

There are several ways to excite a string: f.i. plucking, hitting or bowing.
At a first glance you can consider the excitation as a time-varying function applied to a specific point x_β of the string in the wise of a force, or a boundary condition for the speed or the displacement y.
A more realistic approach requires the consideration of the interactions between the excitation function and the string. This is the case of the exciting behavior of the bow. In our model we consider as excitation functions the pressure and the speed of the bow, and as interacting state variable the location of bowing point x_β, the speed and the acceleration of the bowing point.
The coupled systems is governed by $\sigma(t)$, a Boolean state variable describing the “stick” aut “slip” condition of the bowing point. The bowing point must be one of the sampling points of the string.
Our bow is in some way quite different from that usually implemented by means of continuous speed/force functions. During the stick state, the bowing point x_β is glued to the bow, and we force the speed of that point to be equal to the bow speed. During the slip state, the bowing force F is added to the forces balance in that point. The bowing force depends on the relative speed, in the way shown in the figure underneath.
As you can see, the “stick”→“slip” transition has a threshold that is twice the “slip”→”stick” transition (which happens for zero relative speed).



State diagram of the bowing point. Acceleration (force / linear density) versus point-bow relative speed.

The effect of the bowing pressure is to linearly scale the force (both threshold and continuous curve). One may explore other kinds of dependency.
We don't take explicitly into consideration the temperature of the point - a variable affecting the fluidity and thus the friction behaviour of the rosin (a natural polymer with a complex behaviour).
The shown threshold emulates however the cooling effect of the rosin during the stick time, during which the dissipation is zero. The noise due to the rubbing of the bow over the string is modelled by means of white noise added to the F curve, in such a way to preserve the curve as a "maximum value" of the random value. This is a true "noise model", and adds parametric noise to the model, which imparts a "chaotic" behaviour due to the random interaction between bow and string

3 THE SOUND

The sound is taken as the displacement of the spatial sampling point nearer to the bridge. One can think of this point as a "secant" approximation of the tangent to the string in the bridge position (i.e. the first spatial derivative in the origin), being the tangent the expression of the strain against the bridge.

4 THE METHOD OF CALCULUS

To obtain a continuous model of the shape of the string, we decompose it into a series of sinusoids of spatial frequency, which are multiples of that corresponding to twice the string length.

$$y(x) = \sum_{n=1}^{N} a_n \cdot \sin(n \cdot \pi \cdot x)$$

Thus we imagine the string as periodically and antisymmetrically infinitely expanded in the space. The series is truncated to the first N terms, which represents the maximum spatial frequency allowed to the string. For the free evolution, this limits to N also the partials of the sound. In order to derive the coefficients a_n of such series (i.e., to set-up a continuous model of the string shape) we must know the position of the string in N distinct points - the N spatial sampling points:
Given: $(x_1,y_1),(x_2,y_2) \ldots (x_N y_N)$
One can write:

$$y_i = \sum_{n=1}^{N} a_n \cdot \sin(n \cdot \pi \cdot x_i)$$

$$\vec{y} = \vec{a} \cdot M$$

Where M is a matrix whose elements are:

$$M_{i,n} = \sin(n \cdot \pi \cdot x_i)$$

So that:

$$\vec{a} = \vec{y} \cdot M^{-1}$$

Thanks to the knowledge of an analytic form of the solution, we can easily derive its differential properties in the unit interval; in particular we can formally get the curvatures (i.e. second spatial derivatives) in each of the sampling points.
Leaving out the boring algebra, the calculation of the curvatures can be performed through the product of a matrix (which is a function of the sampling points abscissas) by the positions vector *y*.

$$\vec{y''} = \vec{y} \cdot M'$$

Knowing the curvatures means breaking the equation of the string into N, second order, independent, motion equations of the sampled points. The left term (acceleration) results from the curvature, the velocities (which appear in the term of viscous friction with air) and from the time variation of the curvature itself (term of internal viscous friction). Our today software computes the motion of each point by means of a finite difference method, with an oversampling factor of 4, to reduce approximation errors.

5 RESULTS

The model has been widely tested and used with N=16 and a four times oversampling factor. In these conditions, it runs near to real time on a Pentium 266 system. Current researches are made on better methods of integration, in order to reduce the oversampling factor, reaching thus the real time on today's commercial systems. Research directions are also toward a global reduction of the complexity, to improve the number of harmonics generated, and to reach some degree of polyphony. The bowing point can be made continuous by a technique similar to polyphase filtering, thus requiring memory but no further computational complexity. In the same way can be also implemented supplementary bonds along the string, f.i. slight fingerings like those used to produce harmonics.
As to the excitations, we implemented simple models of pinch and percussion that brought very satisfactory results from the standpoint of imitation. As to the bowing, we made simulations with the specific goal to imitate sound emissions of actual instruments and they gave encouraging results in the case of little variations of the pitch, particularly as to the bow attack transients and the evolution of long-lasting sounds. Furthermore, the sounds generated with our model approximate very well, from an acoustic standpoint, the actual ones, especially in the variation of pressure, velocity and position also in non-standard conditions of execution.

The model has been used with "impossible articulations" (see [9]) not only in the sense that sounds with parameter variations timings that can't be performed by humans have been produced, but also in the extended meaning that parameters, which are physically unreachable, like f.i. internal friction, have been varied during sound emission.

6 CONCLUSIONS

The evaluation of the various direction of research strongly depends on one's goals.
If we assume the imitation of actual instruments as a primary goal of the physical model simulation, we probably know how to interpret the results obtained. But if our goal is instead to provide a new synthesis instrument, both better controllable and more stimulating when compared to other methods, then the evaluation criteria become more complex and finally depend strongly on musical considerations.
In this latter case the imitation of actual instruments is relevant but only in an indirect way. It is just a guideline to verify the correctness of the hypothesis and the methods adopted, but is no longer, by its own, an interesting goal.
From the point of view of contemporary music, the modeling and implementation of features and behaving that are physically impractical may result more important and interesting even if unverifiable.
If we assume this point of view, the key of the evolution of our research lies in the hands of musicians and composers rather than in those of the researchers. Thus it's clear that such a research can be led but through a strong interaction between researchers and musicians.

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